Inequality
https://www.linkedin.com/groups/8313943/8313943-6442730325431984133
Prove that, for any natural number n ≥ 5, the inequality
n/(n + 1) + n/(n + 2) + ... + n/(2n) > 1 +1/2 + 1/3 + ... + 1/n, holds.
Solution by Arkady Alt, **San Jose**,**California**, **USA**.

Note that the inequality holds for $n = 3$ $\left(3\sum_{k=1}^{3}\frac{1}{3+k} - \sum_{k=1}^{3}\frac{1}{k} = \frac{1}{60}\right)$

Let $h_n := \sum_{k=1}^{n}\frac{1}{k}$. Then $\sum_{k=1}^{n}\frac{n}{n+k} = n(h_{2n} - h_n)$ and

$\sum_{k=1}^{n}\frac{n}{n+k} > \sum_{k=1}^{n}\frac{1}{k} \Leftrightarrow n(h_{2n} - h_n) > h_n \Leftrightarrow nh_{2n} > (n+1)h_n.$

Let $a_n := nh_{2n}, b_n := (n+1)h_n$. First we will prove that for any $n \in \mathbb{N}$ holds inequality $a_{n+1} - a_n > b_{n+1} - b_n$.

We have $a_{n+1} - a_n = (n+1)h_{2n+2} - nh_{2n} = h_{2n+2} + n(h_{2n+2} - h_{2n}) =$
$h_{2n+2} + n\left(\frac{1}{2n+1} + \frac{1}{2n+2}\right), b_{n+1} - b_n = (n+2)h_{n+1} - (n+1)h_n =$
$h_{n+1} + (n+1)(h_{n+1} - h_n) = h_{n+1} + 1$. Then $a_{n+1} - a_n > b_{n+1} - b_n \Leftrightarrow$
$h_{2n+2} + n\left(\frac{1}{2n+1} + \frac{1}{2n+2}\right) > h_{n+1} + 1 \Leftrightarrow$
$h_{2n+2} - h_{n+1} + n\left(\frac{1}{2n+1} + \frac{1}{2n+2}\right) > 1$, where latter inequality holds for any $n \in \mathbb{N}$ because
$h_{2n+2} - h_{n+1} + n\left(\frac{1}{2n+1} + \frac{1}{2n+2}\right) > \frac{1}{2n+1} + \frac{1}{2n+2} + n\left(\frac{1}{2n+1} + \frac{1}{2n+2}\right) =$
$\frac{n+1}{2n+1} + \frac{n+1}{2n+2} > 2 \cdot \frac{n+1}{2n+2} = 2 \cdot \frac{1}{2} = 1.$

Since $a_3 > b_3$ and for any $n \geq 3$ assuming $a_n > b_n$ we obtain
$a_{n+1} = (a_{n+1} - a_n) + a_n > (b_{n+1} - b_n) + b_n = b_n$ then by Math Induction,
proved $a_n > b_n, \forall n \geq 3$. Thus $nh_{2n} > (n+1)h_n$ for any $n \geq 3$